

# 算术乘法定义方式的推广

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## Generalization of the Definition Method of the Arithmetic Multiplication

### Abstract

According to Van der Waerden, the sum of two natural numbers is defined as

$$x \cdot 1 = x$$

$$x \cdot y^+ = x \cdot y + x \quad (\text{for every } x \text{ and every } y)$$

The product of two natural numbers is recursively defined as

$$x \cdot 1 = x$$

$$x \cdot y^+ = x \cdot y + x \quad (\text{for every } x \text{ and every } y)$$

$x^+(y^+)$  here means the successor(consequent) of  $x(y)$  in the set of natural numbers. (*Algebra*)

Apparently, the *product* here uniquely depends on how the *sum* works. The generalization of this *product* is thus based on the substitution of addition, the *sum* here. Then the question arose. What result will emerge if we change the addition of natural numbers into a normal arithmetic binary operation?

Inspired by the substitution of original addition, we are going to define an order of arithmetic binary operation in the following text (Sometimes we call the original addition of natural numbers of 1-order, and therefore the multiplication of 2-order and so on. Actually it is an instinctive perception rather than a well-defined mathematical theory. However, to change the addition into a binary operation and generalize the multiplication provides a tool to clarify this so-called order. If we call one binary operation of  $i$ -order, we may treat it as an addition and hence define a multiplication according to the mentioned recursive definition and then call it  $(i+1)$ -order. By using the same method, we can define an operation of  $(i+2)$ -order based on the former  $(i+1)$ -order one.) and then concentrate our attention on the properties of the operations we create by using the generalized multiplication definition.

What is really surprising is if the order of a binary operation is equal to or larger than 2, the commutation, association, and a particular distribution of the binary operation require one same property of its lower-ordered binary operation, and this property leads the lower-ordered operation to be of exactly 1-order. The bulk of the following text is composed of these theorems and their extrapolations.

Furthermore, a discussion on how the demonstrated results can work on the sequence of objects (not only natural numbers) would be presented in the last paragraph.

The motivation of creating this whole paper is from pure mathematical curiosity. Thereby, the whole text is, at least presently, lack of applications.

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