Coordinate-free characterization of homogeneous polynomials with isolated singularities

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November 26, 2009

Abstract

The Durfee conjecture, proposed in 1978, relates two important invariants of isolated hypersurface singularities by a famous inequality; however, the inequality in this conjecture is not sharp. In 1995, Yau announced his conjecture which proposed a sharp inequality. The Yau conjecture characterizes the conditions under which an affine hypersurface with an isoalted singularity at the origin is a cone over a nonsingular projective hypersurface; in other words, the conjecture gives a coordinate-free characterization of when a convergent power series is a homogeneous polynomial after a biholomorphic change of variables. In this project, we prove that the Yau conjecture holds for $n = 5$. As a consequence, we have proved that $5!p_g \leq \mu - p(v)$, where $p(v) = (v-1)^5 - v(v-1)\dots(v-4)$ and p_g, μ, and v are, respectively, the geometric genus, the Milnor number, and the multiplicity of the isolated singularity at the origin of a weighted homogeneous polynomial. In the process, we have also defined yet another sharp upper bound for the number of positive integral points within a 5-dimensional simplex.