

Some Upper Bound Formulas For Ramsey Numbers And Their Applications

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November 25, 2009

Abstract

In this paper, we will discuss some upper bound formulas for Ramsey numbers. At first, we will derive a more unified form from two parameter formulas which was obtained by Yiru Huang etc. Then, similar to their methods, we introduce two new bound formulas for Ramsey numbers in this paper. At last, as the applications of our new bound formulas, some upper bounds for small Ramsey numbers will be obtained.

1 Introduction

1.1 Ramsey numbers

It is well known that of any six persons in the world, three of them must be either acquainted to each other or totally unfamiliar with. Somehow, there are lots of other similar interesting cases, like of any nine persons in the world, three of them should know each other well, while four of them are strangers; of fourteen people, three of them must know each other, while five are strangers. Now let us generalize these cases to find out the rules behind them.

Let $p \geq 2$, $q \geq 2$ be two given positive integers, define $R(p, q)$ as the minimum value of integer n which satisfies the following statement: if we randomly color each side of an n order's complete graph with red or blue, then there must be a wholly red complete graph of p order K_p or a totally blue complete graph of q order K_q . Such $R(p, q)$ is called the Ramsey number of K_p and K_q .

Here are some Ramsey numbers: $R(3, 3) = 6$, $R(3, 4) = 9$, $R(3, 5) = 14$. And a Ramsey number meets two qualities: 1. For any positive integer $p \geq 2$, $q \geq 2$, we have $R(p, q) = R(q, p)$. 2. For any positive integer $q \geq 2$, we also have $R(2, q) = q$.

So far, only a few Ramsey numbers have been discovered. Except for $R(2, q) = q$, we can easily find out some rather small Ramsey numbers, such as $R(3, 3)$, $R(3, 4)$, $R(3, 5)$,