

# Invertibility Probability of Binary Matrices

## Team Members

Angela Dai Caroline Kim John Kim

Coach

Dr. Guofang Wei

School

Dos Pueblos High School

## Abstract

Motivated by an extra credit problem from our Linear Algebra class, we study the invertibility probability of binary matrices (the number of invertible binary matrices divided by the total number of binary matrices). Binary matrices are of interest in combinatorics, information theory, cryptology, and graph theory. It is known that the invertibility probability of  $n \times n$  binary matrices goes to 1 as  $n \rightarrow \infty$ . We conjecture that this probability monotonically increases as the size of the binary matrix increases, and we investigate this by exploring how  $n \times n$  binary matrices of rank  $n$  and rank  $(n-1)$  can be enlarged to  $(n+1) \times (n+1)$  invertible binary matrices. Calculating this explicitly for the identity matrix, we obtain a probable bound that would show that, in a sense, our conjecture is asymptotically true. We also conjecture that as  $n \rightarrow \infty$ , the  $(n+1) \times (n+1)$  matrices produced from  $n \times n$  invertible binary matrices are almost all invertible. We show that this is true on average as well as for almost all  $n \times n$  invertible binary matrices. Furthermore, with the use of a computer, we also computed how many  $(n+1) \times (n+1)$  invertible binary matrices can be enlarged from  $n \times n$  matrices of rank  $n$  and rank  $(n-1)$  for small  $n$ . We study the invertibility probability of matrices with entries in  $\mathbb{Z}_q$  as well.