

# Criteria on Equality of Symmetric Inequalities

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**Abstract:** Algebra inequality can transform to symmetric inequality, symmetric polynomial can be expressed in terms of elementary symmetric polynomials, and so symmetric inequality has become a research focus of inequality. But for a series of problems on the symmetric inequality besides the semi-positive criterion for symmetric forms with six (or more) degrees of  $n$  variables remain unsolved until now. In this paper, we study the criterion on equality and its proof of a class of symmetric inequalities. By controlling two elementary symmetric polynomials and applying the monotone property of functions and Jensen Inequality, we prove some inequalities by the conditions of the criterion on equality, and obtain the necessary and sufficient condition on equality for a class of symmetric inequalities with three variables, then we extend the method to obtain some theorems and corollaries on necessary and sufficient condition on equality for a class of symmetric inequalities with  $n$  variables, and establish criterion theorems for homogeneous symmetric inequalities of degree  $m$  ( $m = 6, 7, 8, 9, 10, 11$ ) with three variables and homogeneous cyclic inequalities of degree four with three variables, we also study the feasibility of establishing criteria for inequalities of higher degrees from theoretical point of view. Thus solve an open problem in the field of inequality which remains unsolved for many years, to start new thought in the research of artificial proof of inequalities, and to provide new basis for the computer proof of inequalities. The paper contains six new theorems and eight new corollaries. At last we give some examples to show its applications in the proof of inequalities.

**Key Words** Inequality, symmetric, cyclic, three-variable,  $n$ -variable, criterion on equality